Magnetic transducer design using a combination of ODE and FEA modeling techniques

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ABSTRACT

The analysis of electromechanical transducers using magnetic drive requires multidomain analysis that includes at least the electrical, magnetic, mechanical domains. Such a system results in a set of differential and algebraic equations (DAE) that can be solved by analogy using modern electrical circuit analysis codes, or with codes written specifically for multidomain DAE modelling. Often, some components in the system require partial differential equations for their analysis, and FEA methods are required. This is especially true in magnetic systems where the flux path including leakage defies simple a priori estimation. The examples of a variable reluctance device is shown.

Keywords: Combined modeling, magnetic transducer, variable reluctance

1. INTRODUCTION

The analysis of electromechanical transducers using magnetic drive requires multidomain analysis that includes at least the electrical, magnetic, mechanical domains. Some types of magnetic transducers can be analyzed using the multiphysics modeling capabilities of modern finite element analysis (FEA) codes. Magnetostrictive materials, however, are not routinely available in commercial codes. While it is possible to approximate their behavior with a linear piezomagnetic approximation in at least one FEA code, this approximation cannot calculate the nonlinear strain vs. field behavior nor can they determine the change in performance with changes in bias magnetic field.

In a research setting, a nonlinear magnetostrictive material model has been developed for use with FEA. However the computational intensity of these methods makes them unattractive for use in initial concept and tradeoff studies in which several design approximations must be compared to determine which is suitable for further more detailed analysis and prototyping. At least in these early stages of design, simpler models based on ordinary differential equations (ODE) can provide an approximate comparison of competing designs.

Especially when analyzing a magnetic system, however, it is challenging to estimate effects such as magnetic leakage and the magnetic reluctance in a complex shaped magnetic return path to be included in the ODE model. In these cases, an FEA model of a part of the structure can determine the magnetic path parameters to be included in the ODE model for improved accuracy.

1.1 Computational Methodology and Tools

There are many suitable tools that can be used for both the ODE calculations and the FEA calculations. The ODE calculations presented in this paper were made using the freely available LTspice1 program. Other versions of SPICE are known to work as well. The FEA calculations were made using the COMSOL multiphysics FEA code2.

2. ANALYSIS OF MAGNETIC SYSTEMS

When a mechanical system is driven by magnetic forces, those forces originate from either the fields in a magnetic gap, or the fields within a magnetostrictive material. Transducers using these two types of force are often called surface force transducers and body force transducers according to the origin of the mechanical force that couples the magnetic and mechanical domains. Analysis methods for the two categories of force is given below in this section.

Features in common to all magnetically operated transducers are bias fields from permanent magnets, bias and/or drive fields from coils, and high permeability magnetic flux paths designed to confine the magnetic fields to a known low
permeability path to minimize magnetic radiation that would otherwise cause energy loss and interference with nearby systems.

In this paper the variables used to model the magnetic are magnetomotive force, \( F \), defined as

\[
F = \int H \, d\ell
\]  

(1)

and magnetic flux \( \phi \) defined as

\[
\phi = \int B \, dA
\]  

(2)

When the flux is conserved and uniform in a segment of the magnetic circuit, these variables are related by

\[
F = \phi R(\phi)
\]  

(3)

where \( R(\phi) \) is called the magnetic reluctance which, in general, is a function of the flux magnitude. In many designs, however, the return path is designed so that the flux is low enough that \( R \) is constant in operation, and Equation (3) has the form of Ohm’s law in electrical circuits. This is the origin of the term “magnetic circuit” for these devices. For a straight bar of constant cross section of high permeability material, as shown in Figure 1(a), the reluctance is calculated as

\[
R(\phi) = \ell / \mu_0 \mu_r A.
\]  

(4)

For a large number of other simple shapes, as shown in Figure 1, the calculation of magnetic reluctance is available in references 3,4.

Sources of magnetomotive force in a magnetic circuit are coils and magnets. For a coil, the magnetomotive force is

\[
F = NI
\]  

(5)

where \( N \) is the number of turns on the coil and \( I \) is the coil current. A reasonable model for a permanent magnet of constant cross section is

\[
F = H_c \ell - \phi R_{magnet} \text{ where } R_{magnet} = \ell / \mu_0 \mu_{recoil} A
\]  

(6)

where \( H_c \) is the recoil permeability of the charged magnet, \( \ell \) is its thickness in the polarization direction, \( \phi \) is the flux through the magnet in operation in the circuit, and \( \mu_{recoil} \) is the effective relative permeability of the charged magnet in operation, often called the recoil permeability.

One significant challenge in magnetic circuit design is reducing magnetic leakage through the air and nonmagnetic materials of the design. The permeability of magnetic materials may be several orders of magnitude higher than that of nonmagnetic materials. However it is not so much greater that leakage is negligible. For accurate analysis, an estimate of
the leakage reluctances is needed; however the leakage paths and reluctances are seldom well understood a priori. Thus, a magnetic FEA model of the structure including the air gaps and nonmagnetic materials is often useful to estimate the leakage. The magnetic circuit for any can be constructed entirely from reluctances, possibly nonlinear, and the static and dynamic sources of coils and magnets.

2.1 A Split Cylinder Transducer with a Variable Reluctance Magnetic Drive

Figure 2(a) is a prototype built as a cross section of a split cylinder transducer. The cylinder vibrates in a clam-shell mode as the magnetic force causes the gap in the top of the figure to alternate expand and contact. The cylinder is driven by a variable reluctance magnetic driver at its gap. Figure 1(b) is a close-up of the magnetic driver structure and the gaps. The figure caption describes the hardware in greater detail.

![Prototype built as a slice of a split cylinder transducer. The coils and magnetic gap are shown at the top of the ring.](a)

![Detail of the magnetic circuit, showing the air gap in the center immediately surrounded by four permanent magnets. The magnetic return path is shown as the light colored material.](b)

The permanent magnets provide a static bias flux that travels in a roughly circular pattern around the loop. The top magnets are polarized in one direction (e.g., to the right), while the lower magnets are polarized in the opposite direction (e.g., top the left). The two coils are wound in opposite directions so that their magnetomotive forces add going around the magnetic loop. The dynamic MMF of the coils alternately adds to or subtracts from the MMF of the magnets, such that the direction of the flux is always in the same direction around the loop, but the magnitude changes dynamically with the drive signal.

The mechanical force in each gap is attractive with magnitude

\[ F = \frac{\varphi^2}{2\mu_0 A} \]  

(7)

where \( \varphi \) is the flux in the gap and \( A \) is the cross sectional area of the gap. Assuming that the flux in the two gaps is the same, and the alignment of the gaps ensures that their opening is the same, then the force acting to close the gaps is also equal. the two forces add to provide the mechanical force that drives the mechanical circuit.
Figure 3 – Analog circuit model with nonlinear components to model the variable reluctance drive, including saturation in the magnetic return path. The magnetic domain includes the magnets, coils, return path and gaps. The mechanical models the single degree of freedom vibrator and the electrical domain includes a simple amplifier model.

Figure 4 – Calculated data from the FEA analysis of the magnetic structures in the split cylinder transducer. At left is the magnetic flux density with magnets only. Center shows the flux density with coils driven for maximum flux in the gap. Right shows flux density for coils driven for minimum flux in the gap.

The features explained above are all present in the analog circuit shown Figure 3. In the center of the figure, the magnetic domain includes two coils, four magnets and reluctances that model the magnetic return path. The electrical circuit is shown in the left part of the figure, simplified to just an ideal current source with a high output resistance. In the lower right of figure 3 is the analog of the mechanical domain, here modeled simply as a single degree of freedom vibrator with an effective modal mass, compliance and damping.
At the opposite ends of the magnetic domain loop are the circuit components that represent the magnetic gaps. Each of these blocks must provide 1) the magnetic reluctance of the gap in the magnetic circuit, given by Equation (4) and including the fact that the gap dimension changes dynamically so that the magnetic reluctance also changes; and 2) the dynamically changing attractive force from Equation (7).

The dynamic nonlinear circuit model of Figure 3 can be reasonably accurate if the component values are chosen appropriately. However, this is difficult to do a priori. Consequently, a single domain magnetic FEA model was used to determine the values of the magnetic circuit components, and a separate structural FEA model was used to calculate the modal mass and compliance for the mechanical domain.

The magnetic FEA analysis is shown in Figure 4. The color density shows the flux density in the magnetic circuit. At left is the static flux from the permanent magnets alone. The color indicates that the magnets provide a flux density in the gap of approximately 0.6 T. The middle figure shows the case when the dynamic coil current is maximum in the direction to add to the static flux. The gap flux has approximately doubled, and the flux magnitude in the return path has approached saturation through most of the center region. The third picture shows the case when the coil current is maximum in the direction to oppose the static flux. Here the flux in the gap is near zero. This analysis provided sizing information for the magnets and return path materials and dimensions.

Separately a dynamic structural FEA was performed with the shell driven at the magnetic gap surfaces. Data from this analysis was used to calculate the effective mass and compliance for use in the mechanical domain of the ODE model.

Figure 5 shows the calculated output of the ODE model of Figure 3 for the displacement of the gap. (Note that the LTspice output graphs are labeled in mV for the vertical scale, but actually represent mm of mechanical displacement.) This calculation can be done in several minutes per frequency on a 64 bit, 4-core, 2.8 GHz Intel processor machine, which is considerably faster than the magnetic-only FEA analysis shown earlier.

Using COMSOL, it is also possible to do a full, coupled structural and magnetic FEA analysis. The mechanical force at the gap surface drives the mechanical motion of the transducer, and the varying gap displacement affects the magnetic reluctance of the gap. This, in turn, affects the magnetic calculation to determine the flux. This complete coupled analysis was performed with version 3.5a of COMSOL. This calculation includes the magnetic saturation behavior of the return path material, a moving mesh calculation in the region of the gap to properly handle the varying gap dimension in the magnetic domain. A transient displacement plot of the driven transducer is shown in Figure 6. This calculation required approximately two days per frequency to compute.
Figure 6 – Postprocessing output from an FEA analysis of the variable reluctance transducer. The model includes nonlinearities in the magnetic materials and in the dynamic force in the gap, and handles the bidirectional coupling between the magnetic and mechanical domains.

3. CONCLUSIONS

To accurately model the performance of magnetically driven transducers, one must include

- the nonlinear magnetization behavior of the materials in the magnetic circuit
- the nonlinear force vs. drive that occurs in magnetic gaps
- the mechanical excitation of the transducer structure by the magnetically generated force
- the effect of the changing gap dimension on the dynamic magnetic reluctance of the gap

ODE modeling approaches have the advantage of significantly reduced computation time, but the disadvantage of inaccurate knowledge of the values of the values of the ODE coefficients. A full multi-domain finite element analysis allows accurate calculation, but required orders of magnitude greater computation time and memory resources. A combined modeling approach that uses FEA to calculate subsystem parameter values to be used in a structurally complete ODE calculation can provide an overall significant reduction in analysis time. This allows many more iterations of a design in the concept stage and can produce more mature transducer designs in a reasonable time.

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REFERENCES